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[1] We present a mesoscale erosion/deposition model, which differs from previous landscape evolution models equations by taking explicitly into account a mass balance equation for the streamflow. The geological and hydrological complexity is lumped into two basic fluxes (erosion and deposition) and two averaged parameters (unit width discharge q and stream slope s). The model couples the dynamics of streamflow and topography through a sediment transport length function $\xi(q)$, which is the average travel distance of a particle in the flow before being trapped on topography. This property reflects a time lag between erosion and deposition, which allows the streamflow not to be instantaneously at capacity. The so-called ξ-q model may reduce either to transport-limited or to detachment-limited erosion modes depending on ξ. But it also may not. We show in particular how it does or does not for steady state topographies, long-term evolution, and high-frequency base level perturbations. Apart from the unit width discharge and the settling velocity, the $\xi(q)$ function depends on a dimensionless number encompassing the way sediment is transported within the streamflow. Using models of concentration profile through the water column, we show the dependency of this dimensionless coefficient on the Rouse number. We discuss how consistent the ξ-q model framework is with bed load scaling expressions and Einstein's conception of sediment motion.

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1. Introduction

[2] It is not very original to say that predictions of landscape evolution models (LEM) are strongly dependent on the erosion model. The way of finding the relevant equation is actually a balance between the complexity of observed processes, the ability to find manageable parameters, and the simplicity of model formulation. Both later points are requirements to make model predictions easy and thus data understandable in terms of dynamics. The stream power law erosion model, which emerged 20 years ago [*Howard and Kerby*, 1983; *Howard*, 1994; *Whipple et al.*, 1999], is the perfect example of such successful compromise. The quest of the right model (note that "right" does not mean universal, which would be a good news for modelers but may be an infringement to evidences that geomorphic systems are not all physically similar) has the following two main issues: (1) the mathematical formulation of erosion rates, with the aim at covering the main physical processes (such as the sediment tool and cover effect in bedrock incision [*Sklar and Dietrich*, 2004]) and (2) the mechanisms of sediment transfer within rivers. The old debate of detachment-limited versus transport-limited processes focuses most of the scientific activity around the latter issue.

[3] Despite several attempts [*Lave and Avouac*, 2001; *Loget et al.*, 2006; *Snyder et al.*, 2003; *Stock and Montgomery*, 1999; *Tomkin et al.*, 2003; *van der Beek and Bishop*, 2003], no simple model has proven adequate to model long-term river evolution in a variety of context. A typical example of the difficulty to define a unique model is that both simple detachment-limited and transport-limited model can predict the exact same steady state river profile [*Lague et al.*, 2003; *Whipple and Tucker*, 2002]. Even more complex models, which include some elements of the physics of the interaction between saltating grains and bedrock, predict geometries that are not distinct enough from simpler formulations [*Sklar and Dietrich*, 2004; *Turowski et al.*, 2007]. It is now clear that only transient, i.e., out of equilibrium, dynamics analysis can help in reducing the range of valid models. Interestingly, no unique model has emerged from the study of transient dynamics, and in some cases, no known model has performed correctly [*Tomkin et al.*, 2003] to reproduce channel evolution. Whether this general failure to predict long-term channel evolution emerges from neglecting more complex effects such as channel width dynamics [*Lave and Avouac*, 2001; *Turowski et al.*, 2006; *Wobus et al.*, 2006], discharge variability effects [*Lague et al.*, 2005; *Tucker and Bras*, 2000], the role of sediment in bedrock incision [*Sklar and Dietrich*, 2004; *Turowski et al.*, 2007] or bedrock channel hydrodynamics, or simply point out a fundamental

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flaw in the way elementary processes are described in present-day models is not clear.

[4] In this paper, we present a new formulation of the processes of erosion, transport and sedimentation dedicated to the modeling of landscape evolution. Although derived in the context of rivers, this model is sufficiently general to be applied to any kind of particle transport processes where particle detachment and deposition can be individually characterized. By introducing a disequilibrium length for sediment deposition we demonstrate how detachment and transport-limited models can be reconciled into a more general class of topographic evolution models that we call ξ-q model. The notion of disequilibrium length is not new and has been introduced in a similar way in channel evolution [*Davy and Crave*, 2000; *Lague et al.*, 2003; *Loget et al.*, 2006], soil erosion problems [*Hairsine and Rose*, 1992], aeolian transport [*Hersen et al.*, 2002; *Kroy et al.*, 2002], or basic experiments [*Charru*, 2006]. In this work we derive its dependency with flow parameters and demonstrate that it is not constant. We then illustrate the basics consequences of this model on various aspects of landscape evolution (steady state geometry, response to tectonic perturbations) and compare these results with the detachment and transport limited model predictions. We then discuss the complexity level that can be introduced in this model in terms of elementary physics of sediment entrainment, bedrock incision and sediment transport.

2. Erosion/Transport Equation Revisited

[5] The general equations governing the dynamics of erosion and sediment transport are derived from both mass balance considerations [*Howard*, 1994; *Paola and Voller*, 2005] and heuristic laws that describe the impact of flow on erosion and/or transport. The mass balance equations are calculated for each system that is considered as an elementary entity of the general dynamics. In most of LEMs, only the "basement" with its upper interface, the topography, is generally considered. We argue here that we can gain in understanding by considering the following two basic systems: the basement (see definition thereafter) and the stream. In this framework, the exchange fluxes are erosion and deposition, defined as the flux of matter taken from basement to stream (erosion), and conversely (deposition). The interest of this framework stays in a better understanding of the parameterization of both fluxes.

[6] The basement is defined as the matter system that is not moving with streamflow. This includes both bedrock and stable, or slowly moving, sediment layer. This could appear as a loose definition since there is a continuum of velocity between the "fixed" basement and main flow with, in between, rolling sediments that participate in the so-called bed load fluxes. We however consider that the velocity transition between "slow" and intermittent bed load velocity and "fast" hydraulic flow is sufficiently sharp to remove this ambiguity.

2.1. System Variables

[7] Parameterization is the starting point, as well as a key aspect, of any simplified theory. The trick is to find the lumped variables that remain consistent with the theoretical framework, and especially its degree of simplicity, but also that fully define the causality relationships basic to the system dynamics. In the following, the key fluxes are defined as the erosion and deposition rates along the bed interface. The physics of fluvial erosion and deposition would emphasize the distribution of flow velocities, and the velocity variations near the bed surface as key parameters. But the simplified theories generally hide the flow complexity within mean fluxes.

[8] As in most previous theoretical frameworks, the basic equations are thus parameterized by the following variables: h_T, the topography defined as the interface between the basement and "stream" systems; q, the stream discharge per unit width, which is the product of the flow depth h and the average flow velocity v; s, topographic slope; q_S, the sediment river load per unit of river width; c_S, the sediment concentration within river that is the volume of sediment normalized by the volume of water. The q_S, c_S, and q variables are related by the expression

$$q_S = c_S q = c_S v h. \qquad (1)$$

Other definitions are given throughout the text.

2.2. Mass Balance Equations

[9] The total volume of sediment involved in the mass balance is the sum of topographic material ($(1 - \phi)h_T$, where the sediment mass porosity ϕ takes account of density changes) and of the sediment content within river $h_S = c_S h$. The bulk mass balance relates the variations of both these terms to the gradients of river fluxes plus the tectonic input [*Paola and Voller*, 2005]

$$\frac{\partial((1-\phi)h_T + c_S h)}{\partial t} = -\mathrm{div}(\mathbf{q_S}) + (1-\phi)T, \qquad (2)$$

where T is the uplift with respect to a reference frame (generally sea level). This mass balance does not tell anything about the partitioning between topography and stream fluxes, which requires a rheological-like equation. At this point, a very common assumption consists in taking $\mathbf{q}_S$ as a univocal function of water discharge q and topographic slope s. This "capacity" model is statistically supported by flux measurements in alluvial rivers; but generalizing a statistical observation as a formal constrain is obviously a step that deserves being tested out.

[10] A more general expression can be obtained by explicitly considering the fluxes between river and topography. For the topographic system, the mass balance equation writes as

$$\frac{\partial h_T}{\partial t} = \frac{-\dot{e} + \dot{d}}{1-\phi} + T, \qquad (3)$$

where $\dot{e}$ and $\dot{d}$ are the erosion and deposition flux, respectively. Note that both fluxes are expressed as the total volume of sediment eroded or deposited per unit of time and area, which justifies the factor $\frac{1}{1-\phi}$ that transforms sediment volume into topographic variations. A similar equation can be written for the river system, which actually combines both equations (2) and (3). It gives the variations of the sediment flux per unit river bed area in a Lagrangian framework attached to streamflow

$$\frac{D(c_S h)}{Dt} = \frac{\partial(c_S h)}{\partial t} + \mathrm{div}(\mathbf{q_S}) = \dot{e} - \dot{d}, \tag{4}$$

where $\frac{D}{Dt}$ is the Lagrangian derivative.

[11] This "two-phase" description, as well as the explicit identification of both erosion and deposition fluxes (*Ancey et al.* [2006], *Charpin and Myers* [2005], and *Einstein* [1950] among others) differs from the *Bagnold*'s [1966, 1973] view, which uses an empirical expression of the river sediment flux q_s, heuristically derived from considering the work done by fluid flow on bed (see *Abrahams and Gao* [2006], *Barry* [2004], and *Martin and Ham* [2005] among others).

[12] Although apparently straightforward, the system dichotomy that leads to both equations (3) and (4) requires a precise definition of the soil/water interface. The issue arises for bed load flux $\mathbf{q_b}$ that may be considered to operate either below of above this interface, i.e., within the soil or river system, respectively. If the interface is assumed above, the mass balance equation (2) must be modified by adding the term $-\mathrm{div}(\mathbf{q_b})$ to its right-hand side, and $\mathbf{q_b}$ must be defined as a heuristic function of model variables. If the interface is assumed below, $\mathbf{q_b}$ is likely to derive from both $\dot{e}$ and $\dot{d}$ fluxes. We demonstrate in a later paragraph that bed load processes are both physically and mathematically consistent with this latter formulation such that it is not necessary to define a priori $\mathbf{q_b}$.

2.3. Erosion Flux

[13] There is nothing new in this paragraph. As for most of LEMs, we assume that the erosion flux $\dot{e}$ is dependent on both the topographic slope s, and of the water discharge q

$$\dot{e} = \dot{e}(q, s).$$

The exact expression for this equation is discussed at length in many papers (see *Whipple* [2004] for a review of bedrock erosion processes and models), and is still an open question. Within the scope of this paper, we consider that the erosion rate derives from the basal shear stress exerted by the streamflow on bed. By using a couple of heuristic equations (see *Lague et al.* [2005] and *Tucker and Bras* [2000] for a complete derivation), we can justify the classical stream power law type of equation

$$\dot{e} = K\, q^m s^n - \dot{e}_c, \tag{5}$$

where m and n are two dimensionless exponents, K is an erosion efficiency factor, and $\dot{e}_c$ a threshold. Note that the shear stress paradigm leads to a more complex expression $\dot{e} = k_e(\tau(q, s) - \tau_c)^a$, where the shear stress τ is also a power law relationship of q and s. This expression is similar to equation (5) if $a = 1$, and equivalent to if the threshold is small.

[14] Equation (5) depends on the nature of the river bed (plain bedrock, partially alluviated or fully alluvial). In short (this is a very hot issue in geomorphology), for bedrock rivers, the erosion efficiency factor is expected to vary with rock lithology [*Lave and Avouac*, 2001; *Sklar and Dietrich*, 2001], the degree of alluvial cover [*Sklar and Dietrich*, 2004; *Turowski et al.*, 2007], and possibly the amount of transported bed load [*Sklar and Dietrich*, 2001]. The slope exponent n is expected to be either 2/3 or 1 and the discharge exponent m in the range [1/3–1/2] [*Whipple*, 2004]. For alluvial rivers, much less is known on $\dot{e}$ because this flux has rarely been considered by itself with the notable exception of studies that aim at quantifying the onset of sediment transport (see *Buffington and Montgomery* [1997] for a review). Most of the studies focus on the total exchange flux $\dot{e} - \dot{d}$, and deduce $\dot{e}$ from sediment concentration profiles [*Garcia and Parker*, 1991]. For entrainment of sediment in suspension, the formulation is generally more complex than equation (5) or its shear stress equivalent [*Garcia and Parker*, 1991; *Parker et al.*, 2003]. However, we will show that a simple shear stress entrainment law can reasonably predict proposed sediment transport capacity laws.

[15] As our objective is to illustrate the consequences of the notion of disequilibrium length on long-term channel dynamics, we retain equation (5) as the simplest possible formulation for erosion rate; but we acknowledge that this assumption bypasses, for instance, the differences between bedrock and alluvial channels.

2.4. Deposition Flux

[16] Physically, the deposition flux depends on what happens within the stream and more especially on the product of the number of particles and their downward velocity. On average (i.e., averaged over the water column), the deposition flux is

$$\langle \dot{d} \rangle = c_S v_S,$$

with v_S the average net downward velocity of sediment grains. v_s is actually the net settling velocity after turbulent upward momentum is accounted for; it depends on grain size, shear velocity, etc. The previous equation defines v_S as the average of the particle net settling velocity weighted by volume.

[17] The deposition rate that we need is defined at the bed interface

$$\dot{d} = c_S^* v_S = d^* c_S v_S, \tag{6}$$

with c_S^* the sediment concentration at the bed interface, and $d^* = \frac{c_S^*}{c_S}$ a dimensionless number equal 1 if the sediment flux is uniformly distributed through depth. Replacing c_S by $\frac{q_S}{q}$ (equation (1)) leads to

$$\dot{d} = d^* \frac{v_S q_S}{q} = \frac{q_s}{\xi(q)}, \tag{7}$$

where the parameter $\xi(q)$ defined as

$$\xi(q) = \frac{q}{d^* v_S} = \frac{vh}{d^* v_S} \tag{8}$$

is the equivalent of a travel, or saltation, length. Physically it represents the average travel distance of sediment grains within flow from the moment they are eroded until they redeposit. We discuss later its relation with the average step length defined by *Einstein* [1950] and characterized by *Ancey et al.* [2003, 2006].

[18] Because of the q dependency of ξ, this model is referred in the following as the ξ-q model. As a very first hypothesis, we assume $\dot{d}^*$ constant, i.e., $\dot{d}$ proportional to $\langle d \rangle$, which leads to ξ proportional to q. But this assumption will be discussed further in the text.

3. Basic Predictions of the Model

3.1. Nonequilibrium Length

[19] To discuss the physical meaning of parameters, we develop equation (4) in the stationary case (no time change of $c_S h$) for flow (both q and q_s) varying in the direction x. With these conditions, the sediment discharge varies along x as

$$\frac{dq_s}{dx} = \dot{e} - \frac{q_s}{\xi}. \tag{9}$$

Except that $\dot{e}$ and ξ depend on the distance x, the previous equation can be viewed as a first-order kinetic equation with ξ the distance to reach the equilibrium. Here $q_s^{\mathrm{eq}} = \xi\dot{e}$ is the long-term equilibrium state of q_s equivalent to the stream capacity; it fixes whether the mass balance is net erosion ($q_s < q_s^{\mathrm{eq}}$) or net deposition ($q_s > q_s^{\mathrm{eq}}$). The first-order kinetic equation of *Beaumont et al.* [1992] and *Kooi and Beaumont* [1996] is similar to equation (9), except that ξ is assumed constant.

[20] This model thus enters into the category of nonequilibrium models [*Cao and Carling*, 2002], for which the sediment transport is not assumed to be systematically at capacity. Its specificity is that the basic disequilibrium parameter ξ depends on flow, and thus on drainage area.

3.2. Detachment-Limited/Transport-Limited Issue

[21] If ξ is large enough to make the deposition flux much smaller than erosion, basic equations (3) and (7) thus combine into

$$\frac{\partial h_t}{\partial t} = -\frac{\dot{e}}{1-\phi} + T, \tag{10}$$

which is typical for the so-called detachment-limited (DL) equation [*Howard and Kerby*, 1983; *Howard*, 1994; *Whipple and Tucker*, 1999].

[22] In contrast, if ξ is small, the sediment load is everywhere close or equal to its equilibrium value (assuming that sediment supply is large enough) $q_s = q_s^{\mathrm{eq}}$ (which is physically consistent with the fact that ξ is the distance to reach equilibrium). The mass balance equation then becomes

$$\frac{\partial h_t}{\partial t} = -\frac{1}{1-\phi}\ \mathrm{div}(\xi\dot{e}\mathbf{u}) + T, \tag{11}$$

where $\mathbf{u}$ is the flow direction. To demonstrate this, we may imagine the mass balance applied to an elementary along-stream pixel of coordinates [x, $x + \Delta x$]. The topographic increase comes from sediments that are eroded upstream and deposited within the pixel. Since ξ is the travel distance of sediments, only sediments located on average at a distance less than ξ from the upstream boundary are going to deposit within the pixel. A similar reasoning is applied to sediments that come out of the pixel, which leads to an expression of the net topographic variation

$$\frac{\partial h_t}{\partial t} = -\frac{1}{1-\phi}\frac{\xi\dot{e}|_x - \xi\dot{e}|_{x+\Delta x}}{\Delta x} + T,$$

equivalent to equation (11) in the limit when Δx approaches 0. Equation (11) is a typical expression for the so-called transport-limited (TL) equation, in which the sediment flux $q_s^{\mathrm{eq}} = \xi\dot{e}$ is

$$q_s^{\mathrm{eq}} = \frac{q}{d^* v_S}(Kq^m s^n - \dot{e}_c). \tag{12}$$

In a model where ξ is taken constant, the slope-area relationship exhibits detachment-limited behavior at short distance and transport-limited at large distance [*Whipple and Tucker*, 2002], with a transition area proportional to the square root of the drainage area A (actually the transition occurs when $\frac{A}{W} = \xi$, with W the river width).

[23] In the ξ-q model, ξ is small for small drainage areas and large for large basins. But it does not mean that the system goes from transport-limited regime on the drainage divide to detachment-limited regime at river mouth since it depends on the reference taken to define the notion of small and large. This aspect is discussed below.

3.3. Physical Meaning of the Stream Capacity

[24] The relationship between stream capacity and erosion rate ($q_s^{\mathrm{eq}} = \xi\dot{e}$ developed in equation (12)) makes the link between different concepts used in the physics of erosion/transport, the erosion of a resting pile of sediment in the one hand, and the sediment transport in the other hand. Using equation (5), the typical expression is

$$q_s^{eq} = \xi(q)K(\tau - \tau_c)^a.$$

For identical flow characteristics (unit discharge and bed shear stress), the stream capacity increases with transport length but also with the bed erodibility K (as statistically, particles will spend less time on bed, and more in suspension/saltation). This underlines the notion that stream capacity emerges from a balance between bed erosion and deposition. Later in this paper, we discuss the link between existing bed load transport formulae and the ξ-q formulation by looking in greater detail at the significance of ξ in the case of saltating grains.

3.4. Dynamic Implications: Steady State and the Slope-Area Relationship

[25] We investigate the steady state topography that results from the ξ-q model. Steady state is defined as a dynamic equilibrium, for which both erosion and deposition rates keep pace with uplift rates. The slope-area relationship at any point P of topography is derived from the mass balance equations (3) by considering that both topography h and unit river outflow q_S are stationary

$$T = \frac{1}{1-\phi}\left(\dot{e} - d^*\frac{v_S q_S}{q}\right). \tag{13}$$

At equilibrium, q_S is the total upstream eroded material $(1-\phi)AT$ divided by the river width W, where A is the drainage area of the basin whose outlet is P

$$q_S = \frac{(1-\phi)AT}{W}.$$

The unit width discharge is likewise related to the upstream rainfall rate r that effectively contributed to discharge (also called effective rainfall rate thereafter)

$$q = \frac{rA}{W},$$

entailing that the ratio $\frac{q_S}{q} = \frac{(1-\phi)T}{r}$. Equation (13) then writes

$$T = \frac{1}{1-\phi}\left(\dot{e} - (1-\phi)d^* \frac{v_S T}{r}\right). \tag{14}$$

By replacing $\dot{e}$ by a power law equation (5), we can derive the basic equation for the slope-area relationship

$$q^m s^n = \frac{(1-\phi)T}{K}\left(\frac{d^* v_S}{r} + 1\right) + \frac{\dot{e}_c}{K}. \tag{15}$$

The eventual slope-area relationship is derived by replacing q by the drainage area (by using the empirical equation that links the river width with water discharge, generally $W \sim Q^{0.5}$ if we neglect the potential dependency of W on incision rate, sediment supply and rock lithology [*Ferguson and Church*, 2004; *Lave and Avouac*, 2001; *Turowski et al.*, 2007].

[26] Equation (15) calls for the following comments:

[27] 1. If the net settling velocity v_s is independent of q and s, the deposition term in the right-hand term of equation (15) is independent of the drainage area. Thus the slope-area relationship exhibits a single scaling whatever drainage area, with an exponent close to $-m/2n$. The form of this equation is similar to that of the detachment-limited case. This result contrasts with previous theories that include both detachment-limited and transport-limited processes, which predict two scaling relationships [*Whipple and Tucker*, 2002] at short (detachment-limited) and large distances (transport-limited). Note that these two scaling relationships, although theoretically predicted, has never been justified by data.

[28] 2. Although the disequilibrium length increases with drainage area, this does not correspond formally to a downstream transition from a transport-limited to detachment-limited regime. Because q_s and ξ are both proportional to A/W, the deposition rate (equation (8)) is constant along stream, and thus so is the erosion rate, which is equal at steady state to $T(1-\phi) - \dot{d}$ according to equation (3). The ratio of deposition over erosion, which is a good indicator of the detachment or transport-limited character of the dynamics, is thus constant along stream.

[29] Equation (15) can be compared with similar expression obtained for the detachment-limited and transport-limited models (equation (12) is taken for the transport-limited model) Detachment-limited

$$q^m s^n = \frac{1}{K}((1-\phi)T + \dot{e}_c)$$

Transport-limited

$$q^m s^n = \frac{1}{K}\left(\frac{v_S d^*(1-\phi)T}{r} + \dot{e}_c\right).$$

All the three erosion/deposition models end up with the same form of slope-area relationships.

3.5. Dynamic Implications: Low-Frequency Evolutions

[30] It is beyond the scope of this paper to explore all the consequences of this erosion/deposition model, in particular when adding some complexity in the erosion term (tool and cover effects, erodability difference along stream between bedrock and alluvium). We just discuss simple cases that can be used as benchmark of erosion laws. The first case is the time required to reach equilibrium for a plateau submitted to a vertical uplift. The erosion/deposition model was compared to both detachment-limited and transport-limited processes defined by equations (10) and (11), respectively. The calculation was performed with the following hypothesis and equations.

[31] The fluvial erosion/transport process is modeled by using equations (3) and (4) for both mass balances. Variables are given in a dimensionless form defined such as the grid mesh, the uplift rate T, and the effective rainfall rate r are all equal to 1. The river width is supposed to vary as the square root of the total discharge, entailing that $W \sim q$ (for simplicity reasons, we take $W = q$, which leads to $q = \sqrt{Q}$).

[32] The erosion equation is a simplified version of (5) $\dot{e} = Kq^{1.0}s = KQ^{0.5}s$. The deposition model is defined by the expression of the disequilibrium length $\xi(q) = \xi_o q = \xi_o Q^{0.5}$. By varying ξ_o, we expect to investigate several behaviors. Note that $\xi_o = 1$ corresponds to a case where ξ is about equal to the flow path length everywhere (the demonstration is similar to that of the Hack's law).

[33] According to equations (10) and (11), both the equivalent detachment-limited and transport-limited models can be encompassed in the same mathematical framework by taking either $\xi = \infty$ for detachment-limited, or $\xi \ll 1$ and equation (12) for transport-limited.

[34] We choose to compare erosion laws that end up to the same eventual topography, and thus to the same slope-area relationships. This was achieved by taking the following erodability constants K for each model: (1) model 1 (ξ-q), $\xi(q) = 10q$ and $\dot{e} = 1.1q^{1.0}s$; (2) model 2 (ξ-q), $\xi(q) = q$ and $\dot{e} = 2q^{1.0}s$; (3) model 3 (ξ-q), $\xi(q) = 0.1q$ and $\dot{e} = 11q^{1.0}s$; (4) model 4 (detachment-limited), $\xi(q) = \infty$ and $\dot{e} = q^{1.0}s$; (5) model 5 (transport-limited), $\xi(q) = 10^{-1}$ and $\dot{e} = 10q^{2.0}s$.

[35] In addition to these fluvial erosion/transport equations, hillslopes are shaped by another process that is responsible for the classical convex/concave hillslope shape. The choice of such a process is rather arbitrary but not critical for this study. Indeed, although hillslope processes control the time scale of system dynamic, it does it by a scaling factor that is independent of fluvial process [*Davy and Crave*, 2000]. Thus the results do not depend on hillslope processes in a relative sense as long as they are the same for all simulations. This assessment has been verified by testing different hillslope laws.

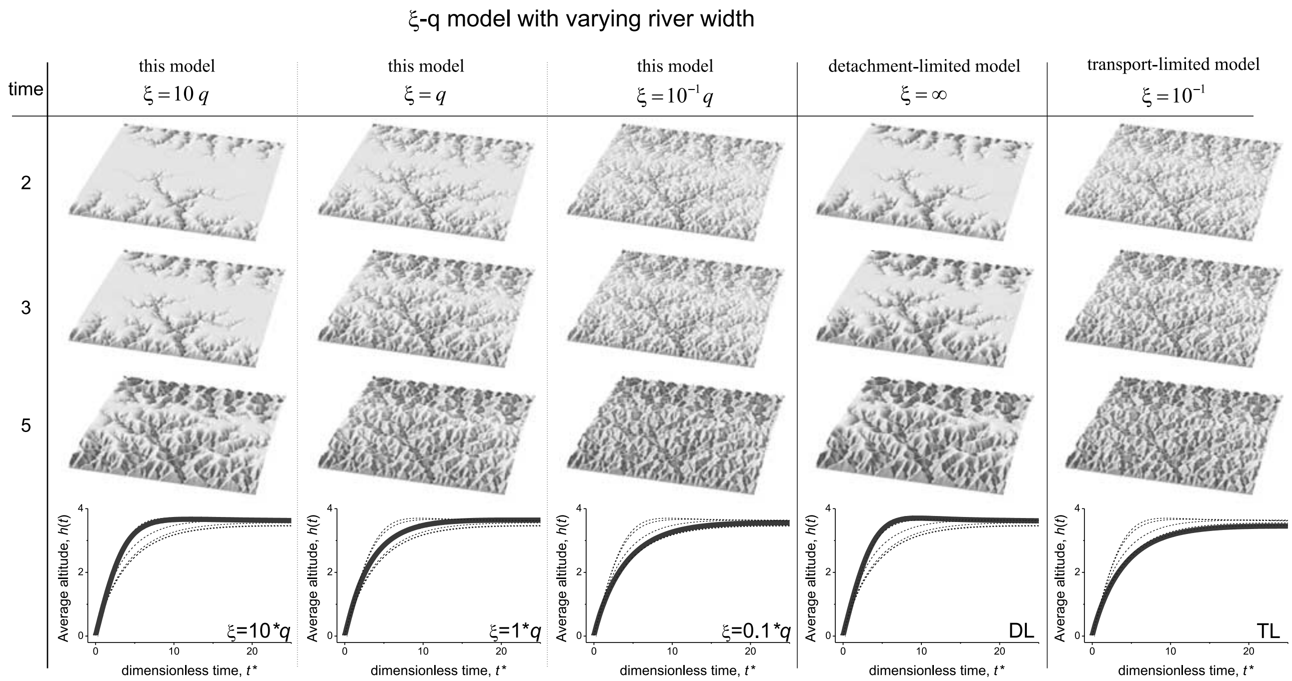

Figure 1. Simulation of the erosion of an uplifted plateau with five different erosion/deposition models. Erosion law as described in the text are shown Topographies at dimensionless times of 2, 3, and 5, respectively, and the average altitude history are shown. Note that the eventual altitude is different from one case to another.

[36] The following hillslope process was chosen for the realism of the eventual topography: a Fickean diffusion with a diffusion coefficient $D = 10^{-4}$, associated with a linear erosion/transfer process corresponding to equation (11) with $q_s = \xi\dot{e} = 3.0 * Q^1 * s^1$. This process operates for water flow less than 10 (in pixel units).

[37] The calculations were performed for a grid of 256 × 256 with the mixed Eulerian-Lagrangian code Eros [*Crave and Davy*, 2001; *Davy and Crave*, 2000; *Lague et al.*, 2003; *Loget et al.*, 2006]. The results of the simulations are shown in Figure 1 for 3 different stages of the topography history. Note that, although the slope-area relationships are similar for all models, the eventual average altitude at steady state can slightly vary because of small differences in the fluvial network organization (Figure 1).

[38] As intuitively expected, the ξ-q model can behave either as its detachment-limited equivalent if $\xi_o > 1$, or as a transport-limited model if $\xi_o < 1$. In the former (this model $\xi = 10q$ and detachment-limited model $\xi = \infty$ in Figure 1), most of the erosion concentrates into large rivers at the first stages, with a fast upstream propagation; steady state is reached much faster than in the detachment-limited case (dimensionless time less than 10). In the latter (this model $\xi = 10^{-1}q$ and transport-limited model $\xi = 10^{-1}$ in Figure 1), the erosion is widespread even at the first stages of process. The case $\xi = q$ is intermediate between both end-members.

[39] For constant ξ (equivalent to the undercapacity model [*Beaumont et al.*, 1992; *Kooi and Beaumont*, 1994]), topography is shaped by detachment-limited-like process at small upstream distance, and transport-limited at large distance [*Howard*, 1980; *Whipple and Tucker*, 2002]. The transition occurs at the point where both processes have the same efficiency (or yields the same slope at equilibrium). Comparing both modes amounts to considering a detachment-limited mode with an erosion rate $\dot{e}$, and a transport-limited mode with a river flux $q_s = \xi\dot{e}$. Equilibrium states between erosion and uplift T are thus defined by the following relationships: $\dot{e} = (1 - \phi)T$ (DL) and $\dot{e} = (1 - \phi)T\frac{q}{r\xi}$ (TL) (see equation (14) and following). The dimensionless number Θ that measures the relative contribution of TL ($\Theta > 1$) and DL modes ($\Theta < 1$) is thus

$$\Theta = \frac{q}{r\xi}. \tag{16}$$

The specificity of the ξ-q model is that Θ is constant along the stream in contrast with the undercapacity model where Θ systematically increases downstream. It means that the very nature of the erosion process (DL-like or TL-like) is similar everywhere. This could explain why no obvious transition of erosion mechanisms is observed in rivers (see section "Field Evidence for Convexity Index of Transport-Limited Erosional Systems" by *Whipple and Tucker* [2002]). This is demonstrated by the simulations presented in Figure 1, which show TL behavior for $\Theta = \xi_o^{-1} > 1$, and DL for $\Theta = \xi_o^{-1} < 1$.

[40] Θ can be related to the ingredients of the physical model by using equations (7) and (16)

$$\Theta = \frac{d^* v_S}{r}; \tag{17}$$

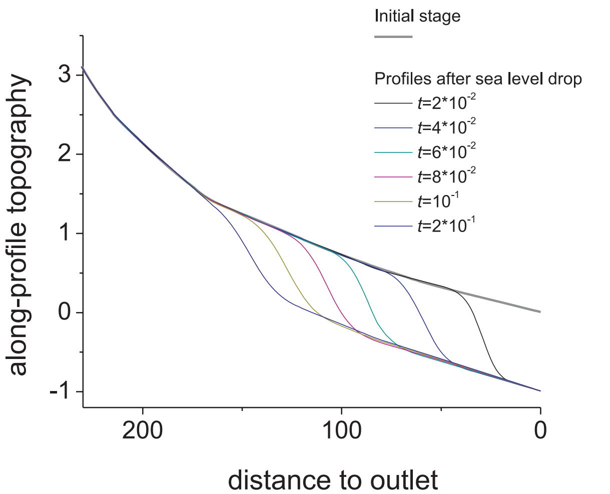

Figure 2. Stream profile for the model 3 of the previous section after applying a lowering of the outlet altitude.

v_S, the volume average vertical velocity of particles in river, takes a large range of values, for example between 10^{-6} and 10^{-1} m s^{-1}. Here r is the effective rainfall, that is the ratio between discharge and drainage area. A reasonable upper bound of its annual average is about 10^{-7} m s^{-1} (corresponding to a rainy climate of 3 m per year), while values about 10^{-5} m s^{-1} (3 cm in a hour) or more are frequently encountered during the main erosive events. This back of the envelope calculation shows that the erosion mode number Θ is generally larger than 1, which is consistent with a transport-limited mode, but detachment-limited mode is likely to occur either for very small particles, or for intense climate event, or if d* is much smaller than 1 (see the discussion thereafter).

[41] The above discussion is valid if the erosion law is similar for bedrock and deposited sediment. This is the case if the basement is a former alluvial system, or if bedrock erosion processes are not that different from transporting rock pieces. But if there is a significant difference between basement and sediment, which is likely to occur in the general case, the previous model at equilibrium can be extended by stating that equilibrium entails both (1) an erosion of the bedrock at the same rate as the uplift T and (2) an erosion of the whole sediment cover. Of course, basement and sediment erosion cannot occur at the same time, which implies a time partitioning of erosion processes between both processes. If α is the percentage of time during which basement erosion occurs at a rate $\dot{e}_B$, and thus $1 - \alpha$ for cover erosion at a rate $\dot{e}_S$ previous statements (1) and (2) lead to the following equations: (1) bedrock erosion (assuming that bedrock porosity is nil) $\alpha\dot{e}_B = T$ and (2) cover erosion $(1 - \alpha)\dot{e}_S = (1 - \phi)\frac{q_S}{\xi} = (1 - \phi)T\frac{q}{r\xi} = (1 - \phi)T\Theta$. Here α can be removed by injecting the former equation in the latter, which leads to

$$\dot{e}_B = T\left((1-\phi)\frac{\dot{e}_B}{\dot{e}_S}\Theta + 1\right).$$

A slope-area relationship can derived from previous equation by replacing $\dot{e}_B$ and $\dot{e}_S$ by their expressions. The complete derivation can be quite complicated and will be done in a further work. The relative contribution of TL and DL modes is given by the dimensionless ratio

$$\Theta' = (1-\phi)\frac{\dot{e}_B}{\dot{e}_S}\Theta.$$

Here Θ' is the product of a large number (Θ) and a small one ($\frac{\dot{e}_B}{\dot{e}_S}$). Given the difficulty in quantifying erosion laws, it remains difficult to determine whether Θ' is smaller or larger than 1, and the issue DL versus TL is still open.

[42] The time partitioning that we invoke for cover and bedrock erosion addresses directly the issue of flow variability whose consequences have yet to be derived in the framework of this model. Since this can lead to quite long developments, we leave this issue for future work.

3.6. Dynamic Implications: High-Frequency Transient State

[43] We suspect that the response of the ξ-q model to fast perturbations (such as very fast base level drop) can be significantly different from both end-member models. We have argued in the previous paragraph that the long-term evolution of ξ-q models (model 3 in Figure 1, for instance) can be equivalent to transport-limited model. Conversely, we suspect that the large disequilibrium length ξ encountered downstream should produce detachment-limited-like evolutions at least during a short period.

[44] To illustrate this dual behavior, we modify the boundary conditions of the topographies eroded during the 5 experiments described in the previous section, by decreasing suddenly the altitude of the outlets of drainage basins. Before this base level drop, the topographies are at steady state. Figure 2 gives the changes of the longest stream topographic profile at different times for the model 3 ($\xi(q) = 0.1q$). It shows the inland propagation of the erosion wave subsequent to the base level drop.

[45] The altitude changes during each experiment are plotted in Figure 3 for the detachment-limited (Figure 3e), transport-limited (Figure 3f), and ξ-q cases (Figures 3a–3d) models, respectively. The curve is obtained by subtracting the stream profile at a given time and the initial profile at $t = 0$. The dotted line indicates the initial step conditions. We choose to compare the ξ-q model 3 ($\xi = 0.1q$) with both the detachment-limited and transport-limited equivalent models.

[46] The step boundary condition is expected to propagate inland and to smooth out because of the diffusion part of the erosion/transport equations. The relative contribution of the former (inland propagation) versus the latter (smoothing) is a signature of the erosion mode. The DL model is close to a wave propagating model, while the TL model contains both propagation and smoothing at the very first stages, but rapidly establish into a diffusion-like regime (Figure 3).

[47] As expected the ξ-q model behaves as DL for $\Theta < 1$ (see above for a definition). On the other hand, the ξ-q model with $\Theta = 100$ has the expected TL-like behavior. But a difference with the long-term response can be observed

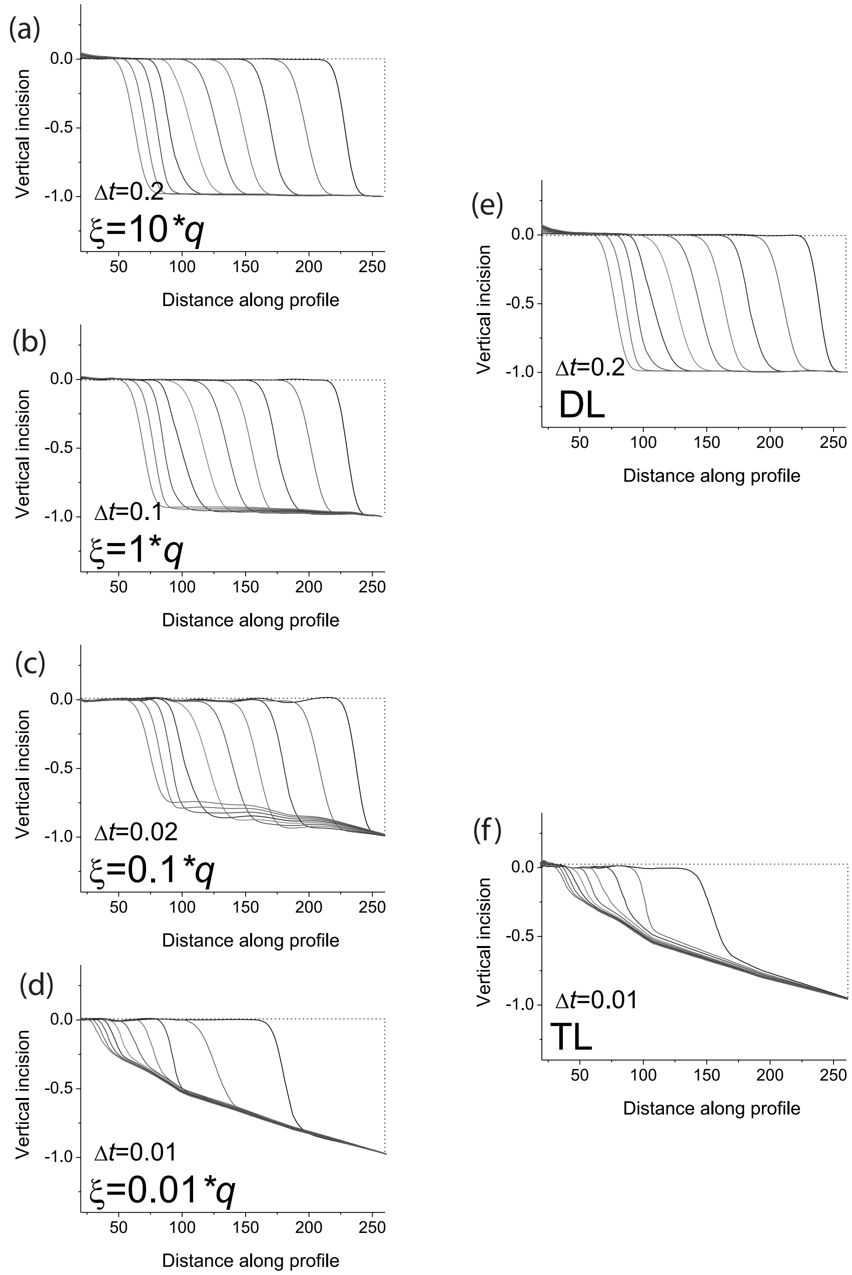

Figure 3. Altitude change during a "step" experiment. The curves represent the difference between the initial stage and the evolving topography for different time steps Δt following the instantaneous base level drop. The dotted lines are the initial stage. Note that the time step is different from one experiment to another. The definition of the disequilibrium length ξ is indicated.

for the case $\Theta = 10$ ($\xi_o = 0.1$), which has now some aspects of DL-like behavior for this simulation. We suspect that this discrepancy is related to the absolute transfer distance ξ of the river outlet, i.e., where the perturbation is applied. A DL-like propagation is defined by ξ larger than the distance over which the perturbation propagates. In the simulations, ξ is about equal to 150 ξ_o at the largest basin outlet (i.e., $\frac{150}{\Theta}$). It cannot be considered as negligible compared to the stream length ($\sim$100) if $\xi_o = 0.1$ (Figure 3c). This example shows up that the ξ-q model cannot be fully mapped on to the equivalent DL or TL models, and that typical features of DL dynamics (knickpoints in particular) can be reproduced without being strictly in DL.

4. Discussions About the ξ-q Model

[48] The modeling exercise for a dynamics as complex as sediment transport and channel incision processes is a tradeoff between physical relevance and simplicity ("Everything must be made as simple as possible, but not one bit simpler," quotation attributed to Albert Einstein). The trick consists in reducing complexity into

a few parameters and equations that keep relevance of basic physical processes. With respect to the other landscape evolution models that consider only topography variations, the ξ-q model broadens the physical (and thus mathematical) framework of long-term erosion/transport equations by considering nonequilibrium transport capacity with a "kinetic" parameter that depends on hydraulic variables. This renders the ability of river to transfer sediment over finite distance with consequences on long- and short-term evolution of topography such as illustrated in the previous paragraphs. The physical consistency of the ξ-q model ("... but not simpler") lies in the capacity of subsuming the physics into the heuristic equations that described basic fluxes of erosion $\dot{e}$ (equation (5)) and deposition $\dot{d}$ (equation (6)), with the additional requirement that these equations only involve lumped variables (here q, the discharge per unit width q, and s, the topographic slope).

[49] Actually, the ξ-q model is an extension of Einstein's view of sediment transport [*Einstein*, 1950] where particles are episodically moving in the fluid, and which has been remarkably illustrated in experiments by *Ancey et al.* [2003, 2006]. This is a way to say that saltation-like processes are ubiquitous in transport processes, even for bed load transport at low shear stresses (see experiments from *Lapointe* [1992], for instance), but with a large range of disequilibrium lengths and erosion rates (or resting times).

[50] A saltation-like description of grains contains the following three stages: erosion/abrasion rate of bed, ejection of grains, and particle settling of grains in fluid. The former and latter processes are directly encompassed in the erosion and deposition fluxes, respectively. Grain ejection is not, but it is likely to control the distribution of sediment within flow, which appears in the parameter d^* of equation (6). Here d^* is actually the ratio between the sediment concentration near the river bed interface and the average over the water column; it intervenes in the definition of ξ and in its dependency with q. Actually d^* contains all the physics that describes the distribution of sediment through water column (grain ejection, upward turbulence forces, etc.), and that the ξ-q model is not aimed at describing. In the later paragraphs, we will discuss how river processes can fit into the ξ-q model framework or how we can adapt the model to take account of the diversity of sediment transport processes. We first discuss how d^* can be calculated for suspended load and bed load rivers.

4.1. Estimate of d^* for Natural Rivers

[51] As explained above, the physics of the transport process is encompassed into the following three main parameters of the ξ-q model: the erosion rate $\dot{e}$, d^* that quantifies the distribution of sediments in the water column, and the settling velocity v_S. As a gross approximation, we consider that most of sediment is carried in a bottom layer of the river, whose thickness is h^* and velocity v^*. The sediment concentration of this bottom layer is the ratio between q_s, the unit sediment flux, and $q^* = v^*h^*$, the actual unit water flow in the bottom layer. The deposition rate writes as

$$\dot{d} = c_S^* v_S = q_S \frac{v_S}{q^*},$$

which leads to simple expressions for the saltation length $\xi(q)$ and d^*

$$\xi(q) = \frac{q^*}{v_S}$$

and $d^* = \frac{q}{q^*}$. This simple calculation shows that the dimensionless number d^* defines in some way the dominant transport process.

[52] A proper estimation of q^* requires knowing both the velocity profile and the sediment concentration in river. The sediment concentration profile within turbulent flow is generally described by a competition between the downward settling of particles and the mixing due to turbulence, whose intensity is given by the velocity distribution. *Rouse* [1937] postulates a Fourier-like diffusion for turbulence mixing, which leads to a first-order equation for sediment concentration where the mixing flux balances the settling velocity. The solution for a logarithmic velocity profile is

$$c_s(z) = c_s(a) * \left(\frac{z-a}{h-a}\frac{a}{z}\right)^Z, \tag{18}$$

where z is the depth, a is a reference depth that is considered to be the base of the river system, h the thickness of the water column, $c(a)$ is the concentration at $z = a$, and Z is a dimensionless parameter that depends on the actual settling velocity v_S and on the shear velocity u^* $Z = \frac{v_S}{\beta\kappa u^*}$, with β as a "correcting" coefficient and κ the von Karman's constant likely equal to 0.4. In the genuine Rouse's theory, Z is the Rouse number $Zo = \frac{v_S}{\kappa u^*}$, and was found to fix the relative contribution of suspended, wash and bed load. However, important corrections (with $\beta > 1$) appear to be necessary to take account of complex flow dynamics over rough bed and/or to fit actual measurements (see *Graf and Cellino* [2002] and data therein).

[53] Further improvements to the original Rouse model concern the "diffusivity" term but the basic ideas remain similar. The modified solutions are either similar to equation (18) with correcting terms applied to Z, or different mathematical equations such as the very simple exponential decrease with a typical length scale equal to h/Ro [*Bridge*, 2003].

[54] Equation (18) can be easily mapped into the ξ-q framework since $c_s(a)$ is likely to be the sediment concentration c_s^* that is involved in equation (6). This postulates that turbulence mixing flux (and thus force) vanishes below $z = a$. Even if this assumption is debatable, we can consider the product $c_s(a) * v_S$ as an upper bound of the deposition flux.

[55] The second assumption that we use to derive simple analytical functions is to consider that the sediment flux q_s can be calculated by integrating $c_s(z)u(z)$ from a to h, which amounts to assuming that the contribution of the sediment flux below $z = a$ is negligible. This is a reasonable assumption when suspended load dominates, or if a can be taken small enough (i.e., within the bed load system) to make the assumption right. Both assumptions make the following calculations only indicative of what ξ could be.

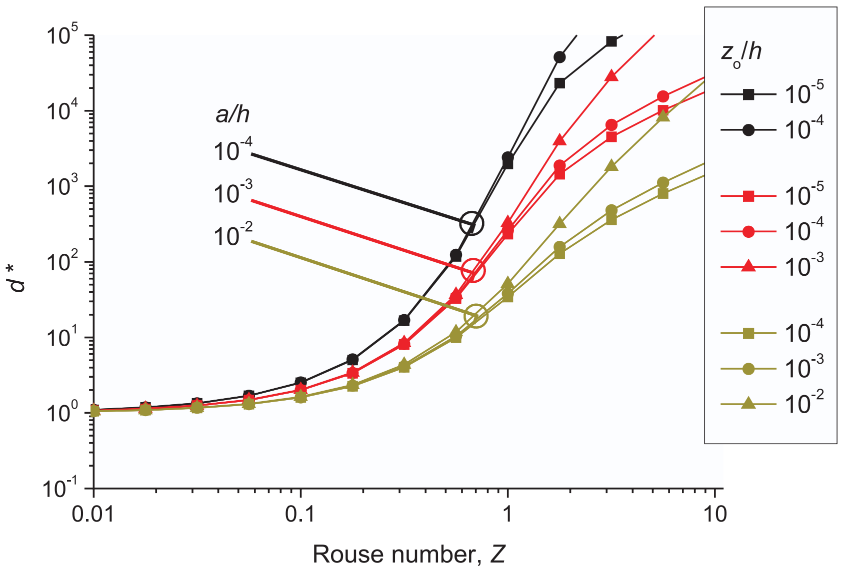

Figure 4. Value of d^* calculated from equation (19) as a function of the Rouse number Z for different values of the ratios a/h and z_o/h. Here d^* is always larger than 1 and smaller than 3 for Rouse numbers Z smaller than 0.1. For Z between 0.1 and 2, d^* increases rapidly mainly as a function of a/h.

[56] The sediment flux can be calculated by integrating $c_s u$ between a and h

$$q_S = \int_a^h c_s(z)u(z)dz = c_S(a)\int_a^h \left(\frac{z-a}{h-a}\frac{a}{z}\right)^Z u(z)dz.$$

We assume $u(z)$ to follow the classical logarithmic velocity profile of turbulent flow, which is besides the basic assumption used to derive the Rouse equation. Then d^* can be calculated from equations (6) and (7)

$$d^* = \frac{c_S(a)}{c_S} = c_S(a)\frac{q}{q_S} = \frac{\int_a^h u(z)dz}{\int_a^h \left(\frac{z-a}{h-a}\frac{a}{z}\right)^Z u(z)dz}$$
$$= \frac{\int_a^h \ln\left(\frac{z}{z_o}\right)dz}{\int_a^h \left(\frac{z-a}{h-a}\frac{a}{z}\right)^Z \ln\left(\frac{z}{z_o}\right)dz}, \qquad (19)$$

with z_o as the characteristic roughness. Here ξ can be calculated from d^* by equation (8). The above expression depends mostly on Z, and only slightly on the ratios a/h and z_o/h (Figure 4).

[57] An example of ξ values calculated from the previous equations is given in Figure 5 as a function of the total discharge for a "representative" alluvial river. This calculation does not pretend to explore all the range of admissible river parameters; it just aims at illustrating a few trends with "reasonable" values. For this, we take the regression parameters given by *Knighton* [1998] for alluvial rivers, where the water depth, river width, and topographic slope scales with the bankfull discharge Q_b as $0.58 * Q_b^{0.36}$, $3.937 * Q_b^{0.5}$, and $0.0007 * Q_b^{-0.3}$, respectively. The shear velocity is derived from its basic definition, $u^{*2} = ghs$, with g the gravity acceleration; with the chosen river parameters, it varies slightly in the range 0.06–0.09 m s^{-1}. Typical settling velocities have been taken from *Dietrich* [1982].

[58] Without pushing too far the previous calculations, we can emphasize the follwoing few points from these graphs. (1) For small rivers and large particles, most of the entrainment mechanism is bed load, d^* is much larger than 1 and the disequilibrium length ξ is small. (2) Conversely, if the Rouse number of large rivers (or of small particle) is small, d^* is close to 1 and the disequilibrium length is defined by the simple relationship $\xi = \frac{q}{v_S}$. (3) Except for coarse sand, the disequilibrium length is about larger than the river width, which may have consequences on the development of meandering or braiding instabilities (this is slightly developed in a next paragraph). (4) The exponential model gives much larger values than the corrected Rouse profile except for fine grains for which d^* is small. Assessing d^* is actually an issue for deriving predictive models. (5) If taking into account a downstream fining, ξ could vary along stream faster than q.

[59] Note that the above simulations are just an example of how ξ can be, and are strongly dependent on the variations of h, W, and s with Q. In the calculations shown in Figure 5, d^* does not vary a lot with Q_b, and thus ξ scales likely as q. But this depends strongly on the along-stream evolution of the shear velocity u^* (and thus of the product $s * h$), and an increase or a decrease of d^* with Q is also likely to occur.

4.2. The ξ-q Model and Bed Load Flux Prediction

[60] For bed load processes, sediment concentrates in an "active" layer whose thickness h^* is small compared to water depth h. However, the exact value of h^*, its dependency with discharge, grain size, etc., is still an issue partly because of the difficulty of having in situ measurements in natural rivers and experiments. Typically, for saltation dominated bed load, the physical definition of h^* is the

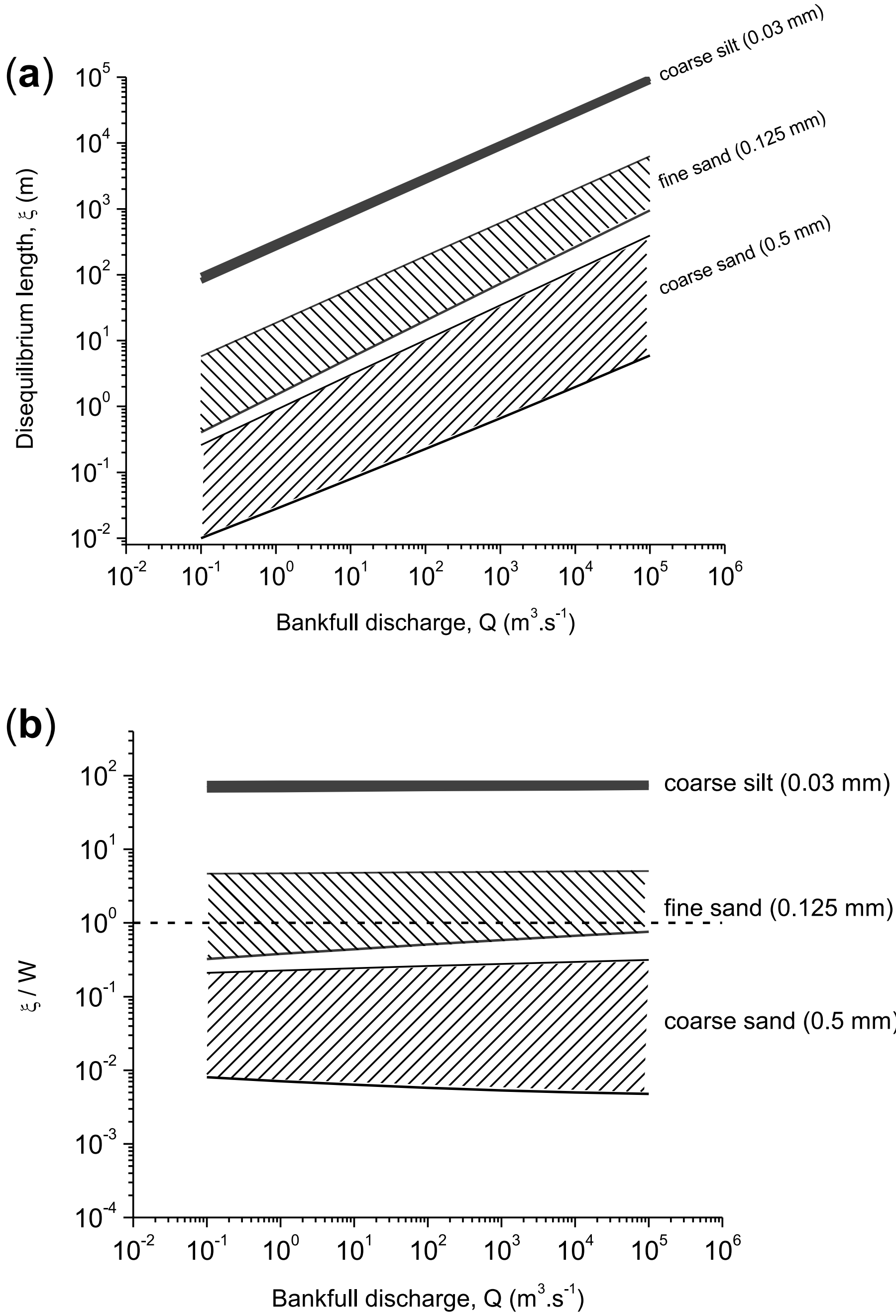

Figure 5. (a) Plot of the disequilibrium length ξ as a function of the bankfull discharge Q_b for the average river of *Leopold and Maddock* [1953]. The settling velocity has been derived from *Dietrich* [1982] for particle diameter of 1 (coarse sand), 0.25 (medium sand), 0.125 (fine sand), 0.062 (very fine sand), 0.03 (coarse silt), and 0.016 mm (medium silt). For each granulometry, the lower curve is calculated with the classical Rouse equation with the correcting term $\beta = 1 + 2(v_S/u^*)^2$ defined by *Graf and Cellino* [2002] and the upper curve is calculated with the exponential concentration profile. (b) Same as Figure 5a but for the ratio ξ by river width.

grain ejection height h^*_e. However, for sheet flow dominated bed load occurring at higher shear stresses [*Gao*, 2008], h^* would likely correspond to the thickness of the layer of actively sheared grains h_s^* located below the active saltation zone. Keeping in mind the difficulty with factoring this complexity into a simple framework, the predicted equilibrium bed load flux can be estimated using $q_\mathrm{b} = \xi\dot{e}$ (equation (12)) and compared to existing bed load transport laws. In this configuration the transport length is replaced by the following expression:

$$\xi = \frac{q^*}{v_S} = \frac{u^* h^*}{v_S},$$

where u^* is the mean velocity of the saltation layer (resp. sheet flow layer) supposed to be proportional to the shear velocity $\tilde{u} = k_v\sqrt{\tau/\rho}$, where k_v is a constant. A generic grain entrainment law can be defined

$$\dot{e} = k_e(\tau - \tau_c)^a,$$

where k_e could incorporate a series of factors such as median grain diameter, sediment concentration (that could have a positive or negative effect on grain entrainment) or the degree of near-bed turbulence. According to equation (11), the equilibrium sediment flux writes as

$$q_b = \xi\dot{e} = k_e\frac{u^*h^*}{v_S}(\tau - \tau_c)^a = \frac{k_e k_v h^*}{\sqrt{\rho}v_S}\tau^{\frac{1}{2}}(\tau - \tau_c)^a. \tag{20}$$

We note that if h^* is constant and $a = 1$, then equation (20) presents the following basic elements of many bed load transport laws [*Bagnold*, 1977; *Einstein*, 1950; *Fernandez Luque and van Beek*, 1976; *Meyer-Peter and Muller*, 1948]: (1) a critical threshold, (2) an asymptotic dependency with shear stress with an exponent 3/2, and (3) an inverse dependency with grain diameter via the sediment settling velocity. Yet, h^* is likely not constant, in the case of saltation-dominated regime, a compilation of particle trajectory analysis [*Sklar and Dietrich*, 2004] shows that the grain ejection height h^*_e varies as $h^*_e \propto D(t - t_c)^{0.5}$, where D is the median grain diameter. Moreover, in the case of coarse gravel, inertial effects induce a positive dependency between the mean settling velocity and shear stress (i.e., the higher is the grain ejected, the more accelerated it can be). Compiled data [*Sklar and Dietrich*, 2004] suggest that $v_s \sim (\tau - \tau_c)^{0.2}$. Combining these results gives a slightly different asymptotic prediction of equilibrium bed load flux if $a = 1$ $q_b \sim \tau^{1.8}$, which is close to the reanalysis of Meyer-Peter and Müller data by *Wong et al.* [2007], or the quadratic model proposed by *Charru et al.* [2004]. For sheet flow dominated regime, h_s^* increases linearly with shear stress [*Gao*, 2008]. However, given the very large sediment concentration and the nonnegligible grain-grain collisions, v_s cannot simply be evaluated. A proper treatment would also require factoring in the impact of sediment concentration on the fluid-grain mixture. This is likely the point where the balance between complexity and efficiency is reached for a model dedicated primarily at landscape evolution modeling.

4.3. The ξ-q Model Versus Einstein's Model

[61] As said before, the ξ-q model shares with *Einstein*'s [1950] model the same conception of episodically moving particles. The point here is to discuss to which extent the physical and mathematical descriptions are similar.

[62] Briefly, Einstein's conception is a particle-based model, where the dynamics is characterized by short flights within fluid and long resting times. The main parameters of this model are the length of the particle flight and the distribution of resting times. The average time spent by a particle within flow can be calculated by considering that the streamflow is a sediment reservoir with a given volume, an inflow, and an outflow. At equilibrium (inflow is outflow), the average residence time of particles is the ratio between volume and outflow. In the river case, the outflow is $\dot{d}$, the volume $c_s h$, and thus the time spent by a particle $t_p = \frac{c_s h}{\dot{d}} = \frac{h}{d^* v_S}$. The average particle distance in the Einstein's model is $\xi^* = t_p v^* = \xi\frac{v^*}{v}$, where v^* is the particle velocity that is likely different from v the average flow velocity. Two additional points: (1) ξ^* has only a statistical meaning and is not by the way a material velocity and (2) the above calculation is not strictly correct since ξ^* is basically the average of the product of time and velocity and not the product of the averages. The above calculation shows that the Einstein's length ξ^* is related to the travel length, but the correspondence is not strictly one to one.

[63] In situ travel lengths have been measured by following particles in sand bed or gravel bed streams with bed load-like entrainment mechanisms [*Church and Hassan*, 1992, 2002; *Habersack*, 2001; *Hassan et al.*, 1991, 1992, 1999, 2006]. The travel distances take values of the order of a couple of meters for gravel bed rivers, and from tenths of meters to couple of kilometers for sand bed rivers. However, apart from *Haschenburger and Wilcock* [2003] who radio-tracked individual particles, they are measured during a flow event or during the flood season, which makes the link with ξ not straightforward. The differences notwithstanding, in situ measurements are at least 1 order of magnitude larger than ξ values calculated with the formula used in Figure 5. For gravel bed rivers, the 6-m average distance measured by *Habersack* [2001] is well above the couple of centimeters that would predict equation (8). This discrepancy addresses two issues. The first is about the actual meaning of ξ in terms of particle displacement, i.e., does it represent only one jump or a trip in the active sheet layer? The second is about the applicability of the Rouse-like theory when dealing with bed load processes. Even in introducing correcting terms such as β in the formulae, it is doubtful that this theory succeeds to fit concentration profile in saltating bed load streams. A better approach would be to consider the particle ejection height, h^*, in the definition of ξ as given by *Charru* [2006], but natural constraints are badly needed.

[64] A similar reasoning can be applied for the particle rest periods. The survey of particle histories, motivated by the Einstein's model, clearly demonstrates that particles move only episodically with quite a large distribution of rest periods including "long-term" deposition. Linking the rest periods with the erosion rate requires assessing the volume of particles within the active layer. It is likely given as the average burial depths of particles when they stop. *Hassan et al.* [1999], for instance, measured depth of the order of several tenths of centimeters for a sand bed river. We can imagine a scour and fill process to be model in the ξ-q model framework in relationship but this would require a detailed modeling of each flood that is beyond the scope of the ξ-q model, as is bed form dynamics that is likely to play a role in the particle displacement.

[65] To conclude with this part, it is not straightforward to match the ξ-q model into a particle-based theory such as *MacVicar et al.* [2006] and *Malmaeus and Hassan* [2002], because it is based on a flux parameterization. Here ξ, for instance, is typically the average distance for stream to reach equilibrium, which is defined as statistic equality between erosion and deposition rates. Whether ξ relates or not to the average particle travel distance is still an issue that deserves being explored.

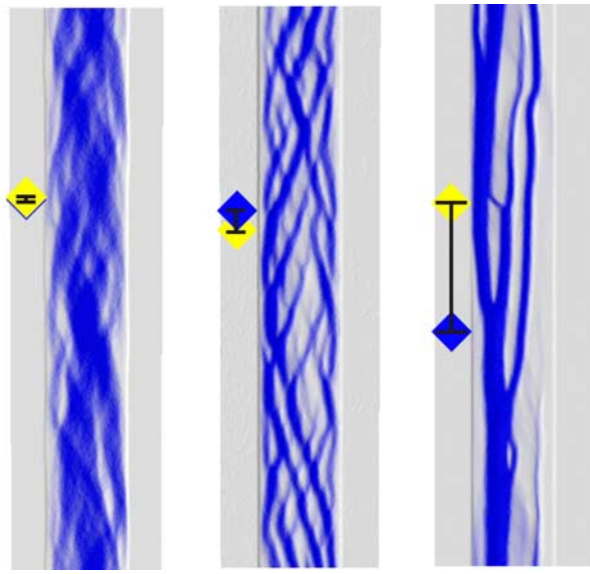

Figure 6. Top views of three simulations calculated from the particle-based model Eros with three different disequilibrium lengths, i.e., ξ = (left) 2, (middle) 5, and (right) 100, respectively. All the other erosion parameters of equation (5) are similar (K = 1, m = 1.5, n = 1). For each view, ξ corresponds to the distance between the blue and yellow squares. The blue color indicates water flow. Note that this version of the Eros program is slightly more sophisticated than that used in Figures 1–3. The river width emerges from dynamics thanks to the following two additional terms: a heuristic description of water height that allows channel avulsion and lateral channel erosion.

4.4. The ξ-q Model and Geomorphic Instabilities

[66] Here ξ is above all a "disequilibrium" distance, meaning that a river with uniform conditions in flow and erosion rates can be out of equilibrium (deposition does not balance erosion) on distances smaller than ξ. Because of this potential discrepancy between erosion and deposition, this length was found to be a major control in the development of dunes or sand ripples in either aerial or fluvial regimes [*Kroy et al.*, 2002]. Here ξ also called the saturation length [*Kroy et al.*, 2002] inhibits the growth of the smallest wavelengths; it is thus found to give the minimal size of dunes [*Andreotti and Claudin*, 2007; *Hersen et al.*, 2002; *Parteli et al.*, 2007a; 2007b]. The physical significance of the "saturation length" in the case of aerial dune is however different than ξ. There, it likely originates in the particle inertia within the airflow. In viscous fluid, inertia is negligible and [*Charru*, 2006] proposed that ξ is a deposition length whose definition is exactly similar to ours.

[67] The understanding of other fluvial instabilities, such as channel bars that are basic to braiding, is not so advanced [*Bridge*, 2003], but still the transfer distance of particles is likely controlling the distribution of bars in braided rivers [*Pyrce and Ashmore*, 2003, 2005]. Although a full treatment of this issue is beyond the scope of this paper, we show in Figure 6 three simulations made with the particle-based Eros code with small and large disequilibrium length ξ that illustrates its effect on braiding. The code is an enhanced version of the one used above and presented by *Crave and Davy* [2001] and *Davy and Crave* [2000], where particles are elementary water volumes representing a full slice of river. Wide channels and braided patterns actually results from the following two additional rules:

[68] 1. In addition to the underlying river bed, particles are eroding their side neighbors (except the upstream and downstream neighbors that are eroded along the flow path) at the same rate as basal erosion. The surface on which lateral erosion applies is the altitude difference between the pixel and its neighbor, and thus the volume eroded laterally is the basal erosion volume multiplied by lateral slope. Deposition is assumed to occur only vertically on the basal bed.

[69] 2. The former code neglected water depth, and routed particles on top of bed topography. However, this assumption precludes channels from overflowing, which seriously limits the number of active channels that form the braided structure. The enhanced code calculates explicitly water depth h from discharge q and slope s via a Darcy-Weisbach-like equation (in these simulations where slope is about constant, the exact nature of the equation, Manning or Darcy-Weisbach, is less important than ξ). Particles are then moved on top of the water surface. A smoothing procedure is applied to the water surface to remove unmanageable roughness due to discharge variability.

[70] Figure 6 shows that a braid-like instability develops if ξ is neither too small (the bed load regime) nor too large (only straight rivers develop). The trade-off for a full braiding system is likely obtained when ξ is a fraction of the braid plain width, but larger than individual channel width. This range of ξ does not seem unusual according to Figure 5. Note that the erosion/deposition law is only part of the braiding issue. The simulations of Figure 6 were performed in a pure sediment transfer regime (exact balance of erosion by deposition at the system scale). A departure from this stationary case inhibits braiding by forming either straight rivers (erosion case) or delta-like patterns (deposition case).The ξ-q model is actually potentially richer in terms of channel forms than a model with constant ξ. All the consequences of the ξ-q model have still to be derived for alluvial and bedrock rivers. In the latter case, refinements will likely be necessary as the flux of depositing particle can potentially play a role in the erosion flux (i.e., the tool effect in bedrock river abrasion [*Sklar and Dietrich*, 2004]) and partial alluvial cover limits bedrock erosion (requiring to specify alluvial thickness above bedrock as a third state parameter on top of unit discharge and channel slope).

5. Conclusion

[71] In this paper, we develop and discuss a mesoscale erosion/deposition model, which differs from previous LEM equations by taking explicitly into account a mass balance equation for the streamflow. This approach is not new, and even less sophisticated than that by *Paola and Voller* [2005], but we try to develop a physically meaningful parameterization for this model, and to derive consequences on relief dynamics.

[72] As for other LEMS, the model is based on a mesoscopic description of the physical processes that reduces the geological and hydrological complexity into average parameters. All the physics is lumped into two basic fluxes,

erosion and deposition, and the following two averaged parameters: unit width discharge q, and stream slope s.

[73] Basically, the model consists in coupling the dynamics of streamflow and topography through a sediment transport length function $\xi(q)$, which is the average distance covered by a particle in the streamflow before being trapped on topography. This property reflects a time lag between erosion and deposition, which allows the streamflow not to be instantaneously at capacity. The disequilibrium distance function is basically proportional to the ratio of q to the net settling velocity.

[74] The ξ-q model (a name that emphasizes the dependency of ξ with q) has the same property as other models based on a disequilibrium length. The product of erosion rate and ξ, $q_S = \xi\dot{e}$, is the stream capacity that is eventually reached along stream when erosion keeps pace with deposition. If ξ is small, the model reduces to a classical transport-limited equation, where $q_S = \xi\dot{e}$ is the equivalent "bed load" flux. If ξ is large, sediments never redeposit after being eroded, and the ξ-q model is typical of detachment-limited behavior.

[75] The consequences of the ξ-q model have been analyzed for long-term geological evolutions as well as for high-frequency changes. In the former case, the model challenges undercapacity models where the disequilibrium length ξ is constant, which predict a transition from DL to TL along stream. Whatever the models, the sediment entrainment mode is given by the dimensionless number $\Theta = \frac{q}{r\xi}$, with r the average upstream effective rainfall rate, with both end-members DL for $\Theta \ll 1$, TL for $\Theta \gg 1$. Undercapacity models with constant ξ predict a downstream increase of Θ, and a consequent transition from DL to TL at the critical discharge where $\Theta = 1$. Conversely, Θ is constant along stream for the ξ-q model where ξ is proportional to q, and the very nature of the erosion process (DL-like or TL-like) does not change along stream. We show that this leads to a unique power law for the slope-area relationships even if large ξ values are encountered downstream. We derive the corresponding equations.

[76] High-frequency evolutions lead to slightly different conclusions mainly because of the consequences of large ξ values that the ξ-q model predicts downstream. Some experiments in the TL-like regime during long-term changes (i.e., with $\Theta < 1$) exhibit DL-like behaviors during high-frequency variations. This was illustrated by the inland propagation of a base level drop, which propagates as a pure wave for these experiments as it does for DL models.

[77] Apart from the unit width discharge q and the settling velocity v_s, the ξ function is likely to depend on a dimensionless number that encompasses the way sediment is transported within the streamflow. By using models of concentration profile through the water column, we show that this dimensionless coefficient depends on the Rouse number Zo, is about 1 for small Rouse number ($Zo < 0.1$), but can become very small for larger Zo.

[78] Finally we discuss how consistent the ξ-q model framework is with bed load scaling expressions, considering that the bed load flux q_b is matched into the product of ξ and erosion rate $\dot{e}$. We also discuss how the model relates with the Einstein's conception of sediment motion, which is based on the description of the motion of individual particles with travel distances within flow and rest periods.

[79] The model is a suitable framework to explore more complex systems. Feedbacks between erosion, sediment transport and deposition (such as tool or cover effects of bedrock channels) can be easily implemented and the consequences tested. The model is also a suitable to explore the consequences of discharge variability on both erosion and transport.

Notation

A	drainage area.
a	height of the base of the turbulent layer, used as the reference depth for the Rouse profile.
c_S	sediment concentration within stream, equal to the volume of sediment normalized by water volume $c_S = q_S/q$.
c_S^*	the sediment concentration at the bed interface.
$\dot{d}$	deposition flux.
d^*	the ratio between c_S, the mean sediment concentration in stream, and c_S^*.
$\frac{D}{Dt}$	Lagrangian derivative with time.
$\dot{e}$	erosion flux.
g	gravity acceleration.
h	flow height.
h_T	topography (defined as the interface between the basement and "river" systems).
K	sediment erodibility (assuming that $\dot{e} = Kq^m s^n - \dot{e}_c$).
q	river discharge per unit width $q = vh$.
q_S	sediment river load per unit of river width.
r	rainfall rate that effectively contribute to discharge (or effective rainfall).
S	topographic slope.
T	tectonic uplift (with respect to a reference frame generally taken as the sea level).
t	time.
u^*	the shear velocity defined by $u^{*2} = ghs$.
v	average flow velocity.
v_S	the net particle downward velocity, which is the net settling velocity after turbulent upward momentum is accounted for.
W	river width.
Zo	the Rouse number, $Zo = \frac{v_S}{\kappa u^*}$, with κ von Karman's constant.
Θ	TL-DL dimensionless number.
ϕ	sediment mass porosity ($1 - \phi$ is the mass ratio between a volume of bed material and a corresponding volume of sediment grains).
ξ	disequilibrium or sediment length equal to the ratio between the sediment river load q_S and the deposition flux $\dot{d}$.

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References

Abrahams, A. D., and P. Gao (2006), A bed-load transport model for rough turbulent open-channel flows on plane beds, *Earth Surf. Processes Landforms*, *31*(7), 910–928, doi:10.1002/esp.1300.

Ancey, C., et al. (2003), Rolling motion of a bead in a rapid water stream, *Phys. Rev. E*, *67*(1), 011303.1–011303.11, doi:10.1103/PhysRevE.67.011303.

Ancey, C., et al. (2006), Statistical description of sediment transport experiments, *Phys. Rev. E*, *74*(1), 011302.1–011302.14, doi:10.1103/PhysRevE.74.011302.

Andreotti, B., and P. Claudin (2007), Comment on "Minimal size of a barchan dune", *Phys.Rev. E*, *76*(6), 063301.1–063301.5, doi:10.1103/PhysRevE.76.063301.

Bagnold, R. A. (1966), An approach to the sediment transport problem from general physics, *U.S. Geol. Surv. Prof. Pap.*, *422-I*, 1–42.

Bagnold, R. A. (1973), The nature of saltation and of "bed-load" transport in water, *Proc. R. Soc. London Ser. A*, *332*(1591), 473–504, doi:10.1098/rspa.1973.0038.

Bagnold, R. A. (1977), Bed load transport by natural rivers, *Water Resour. Res.*, *13*(2), 303–312, doi:10.1029/WR013i002p00303.

Barry, J. J. (2004), A general power equation for predicting bed load transport rates in gravel bed rivers, *Water Resour. Res.*, *40*(10), W10401, doi:10.1029/2004WR003190.

Beaumont, C., et al. (1992), Erosional control of active compressional orogens, in *Thrust Tectonics*, edited by K. R. McClay, pp. 1–18, Chapman and Hall, New York.

Bridge, J. S. (2003), *Rivers and Floodplains: Forms, Processes, and Sedimentary Record*, 491 pp., Blackwell, Oxford, U. K.

Buffington, J. M., and D. R. Montgomery (1997), A systematic analysis of eight decades of incipient motion studies, with special reference to gravel-bedded rivers, *Water Resour. Res.*, *33*(8), 1993–2029.

Cao, Z., and P. A. Carling (2002), Mathematical modelling of alluvial rivers: Reality and myth. Part I: General review, *Proc. Inst. Civ. Eng.*, *154*(3), 207–219.

Charpin, J. P. F., and T. G. Myers (2005), Modelling thin film flow with erosion and deposition, *Adv. Water Resour.*, *28*(7), 761–772, doi:10.1016/j.advwatres.2004.12.010.

Charru, F. (2006), Selection of the ripple length on a granular bed sheared by a liquid flow, *Phys. Fluids*, *18*(12), 121508.1–121508.9, doi:10.1063/1.2397005.

Charru, F., H. Mouilleron, and O. Eiff (2004), Erosion and deposition of particles on a bed sheared by a viscous flow, *J. Fluid Mech.*, *519*, 55–80, doi:10.1017/S0022112004001028.

Church, M., and M. A. Hassan (1992), Size and distance of travel of unconstrained clasts on a streambed, *Water Resour. Res.*, *28*(1), 299–303, doi:10.1029/91WR02523.

Church, M., and M. A. Hassan (2002), Mobility of bed material in Harris Creek, *Water Resour. Res.*, *38*(11), 1237, doi:10.1029/2001WR000753.

Crave, A., and P. Davy (2001), A stochastic "precipiton" model for simulating erosion/sedimentation dynamics, *Comput. Geosci.*, *27*(7), 815–827, doi:10.1016/S0098-3004(00)00167-9.

Davy, P., and A. Crave (2000), Upscaling local-scale transport processes in large-scale relief dynamics, *Phys. Chem. Earth A*, *25*(6–7), 533–541, doi:10.1016/S1464-1895(00)00082-X.

Dietrich, W. E. (1982), Settling velocity of natural particles, *Water Resour. Res.*, *18*(6), 1615–1626.

Einstein, H. (1950), The bed-load function for sediment transportation in open channel flows, *Tech. Bull. 1026*, 78 pp., U.S. Dep. of Agric., Washington, D. C.

Ferguson, R. I., and M. Church (2004), A simple universal equation for grain settling velocity, *J. Sediment. Res.*, *74*(6), 933–937, doi:10.1306/051204740933.

Fernandez Luque, R., and R. van Beek (1976), Erosion and transport of bed-load sediment, *J. Hydraul. Res.*, *14*, 127–144.

Gao, P. (2008), Transition between two bed-load transport regimes: Saltation and sheet flow, *J. Hydraul. Eng.*, *134*(3), 340–349.

Garcia, M., and G. Parker (1991), Entrainment of bed sediment into suspension, *J. Hydraul. Eng.*, *117*(4), 414–435.

Graf, W. H., and M. Cellino (2002), Suspension flows in open channels; Experimental study, *J. Hydraul. Res.*, *40*(4), 435–447.

Habersack, H. M. (2001), Radio-tracking gravel particles in a large braided river in New Zealand: A field test of the stochastic theory of bed load transport proposed by Einstein, *Hydrol. Processes*, *15*(3), 377–391, doi:10.1002/hyp.147.

Hairsine, P. B., and C. W. Rose (1992), Modeling water erosion due to overland-flow using physical principles: 1. Sheet flow, *Water Resour. Res.*, *28*(1), 237–243, doi:10.1029/91WR02380.

Haschenburger, J. K., and P. R. Wilcock (2003), Partial transport in a natural gravel bed channel, *Water Resour. Res.*, *39*(1), 1020, doi:10.1029/2002WR001532.

Hassan, M. A., et al. (1991), Distance of movement of coarse particles in gravel bed streams, *Water Resour. Res.*, *27*(4), 503–511, doi:10.1029/90WR02762.

Hassan, M. A., et al. (1992), Virtual rate and mean distance of travel of individual clasts in gravel-bed channels, *Earth Surf. Processes Landforms*, *17*(6), 617–627, doi:10.1002/esp.3290170607.

Hassan, M. A., et al. (1999), The transport of gravel in an ephemeral sandbed river, *Earth Surf. Processes Landforms*, *24*(7), 623–640, doi:10.1002/(SICI)1096-9837(199907)24:7<623::AID-ESP978>3.0.CO;2-2.

Hassan, M. A., et al. (2006), Experiments on the effect of hydrograph characteristics on vertical grain sorting in gravel bed rivers, *Water Resour. Res.*, *42*, W09408, doi:10.1029/2005WR004707.

Hersen, P., et al. (2002), Relevant length scale of Barchan dunes, *Phys. Rev. Lett.*, *89*(26), 264301.1–264301.4, doi:10.1103/PhysRevLett.89.264301.

Howard, A. D. (1980), Thresholds in river regimes, in *Thresholds in Geomorphology*, edited by D. R. Coates and J. D. Vitek, pp. 227–258, Allen and Unwin, Winchester, Mass.

Howard, A. D. (1994), A detachment-limited model of drainage basin evolution, *Water Resour. Res.*, *30*(7), 2261–2285, doi:10.1029/94WR00757.

Howard, A. D., and G. Kerby (1983), Channel changes in badlands, *Geol. Soc. Am. Bull.*, *94*, 739–752, doi:10.1130/0016-7606(1983)94<739:CCIB>2.0.CO;2.

Knighton, D. (1998), *Fluvial Forms and Processes: A New Perspective*, 383 pp., Arnold, London.

Kooi, H., and C. Beaumont (1994), Escarpment evolution on high-elevation rifted margins: Insights derived from a surface processes model that combines diffusion, advection, and reaction, *J. Geophys. Res.*, *99*, 12,191–12,209.

Kooi, H., and C. Beaumont (1996), Large-scale geomorphology: Classical concepts reconciled and integrated with contemporary ideas via a surface processes model, *J. Geophys. Res.*, *101*(B2), 3361–3386, doi:10.1029/95JB01861.

Kroy, K., et al. (2002), Minimal model for sand dunes, *Phys. Rev. Lett.*, *88*(5), 054301.1–054301.4, doi:10.1103/PhysRevLett.88.054301.

Lague, D., et al. (2003), Laboratory experiments simulating the geomorphic response to tectonic uplift, *J. Geophys. Res.*, *108*(B1), 2008, doi:10.1029/2002JB001785.

Lague, D., et al. (2005), Discharge, discharge variability, and the bedrock channel profile, *J. Geophys. Res.*, *110*, F04006, doi:10.1029/2004JF000259.

Lapointe, M. (1992), Burst-like sediment suspension events in a sand bed river, *Earth Surf. Processes Landforms*, *17*(3), 253–270, doi:10.1002/esp.3290170305.

Lave, J., and J. P. Avouac (2001), Fluvial incision and tectonic uplift across the Himalayas of central Nepal, *J. Geophys. Res.*, *106*(B11), 26,561–26,591, doi:10.1029/2001JB000359.

Leopold, L. B., and T. J. Maddock (1953), The hydraulic geometry of stream channels and some physiographic implications, *U.S. Geol. Surv. Prof. Pap.*, *252*, 1–57.

Loget, N., et al. (2006), Mesoscale fluvial erosion parameters deduced from modeling the Mediterranean sea level drop during the Messinian (late Miocene), *J. Geophys. Res.*, *111*, F03005, doi:10.1029/2005JF000387.

MacVicar, B. J., et al. (2006), A two-dimensional discrete particle model of gravel bed river systems, *J. Geophys. Res.*, *111*, F03009, doi:10.1029/2005JF000316.

Malmaeus, J. M., and M. A. Hassan (2002), Simulation of individual particle movement in a gravel streambed, *Earth Surf. Processes Landforms*, *27*(1), 81–97, doi:10.1002/esp.305.

Martin, Y., and D. Ham (2005), Testing bedload transport formulae using morphologic transport estimates and field data: Lower Fraser River, British Columbia, *Earth Surf. Processes Landforms*, *30*(10), 1265–1282, doi:10.1002/esp.1200.

Meyer-Peter, E., and R. Muller (1948), Formulas for bedload transport, in *Proceedings of the 2nd Meeting of the International Association for Hydraulic Structures Research*, edited by the International Association for Hydraulic Structures Research, pp. 39–64, Int. Assoc. for Hydr. Struct. Res., Stockholm.

Paola, C., and V. R. Voller (2005), A generalized Exner equation for sediment mass balance, *J. Geophys. Res.*, *110*, F04014, doi:10.1029/2004JF000274.

Parker, G., et al. (2003), Bed load at low Shields stress on arbitrarily sloping beds: Alternative entrainment formulation, *Water Resour. Res.*, *39*(7), 1183, doi:10.1029/2001WR001253.

Parteli, E. J. R., et al. (2007a), Minimal size of a Barchan dune, *Phys. Rev. E*, *75*(1), 011301.1–011301.10, doi:10.1103/PhysRevE.75.011301.

Parteli, E. J. R., et al. (2007b), Reply to "Comment on 'Minimal size of a barchan dune,'", *Phys. Rev. E*, *76*(6), 063302.1–063302.5, doi:10.1103/PhysRevE.76.063302.

Pyrce, R. S., and P. E. Ashmore (2003), The relation between particle path length distributions and channel morphology in gravel-bed streams: A synthesis, *Geomorphology*, *56*(1–2), 167–187, doi:10.1016/S0169-555X(03)00077-1.

Pyrce, R. S., and P. E. Ashmore (2005), Bedload path length and point bar development in gravel-bed river models, *Sedimentology*, *52*(4), 839–857, doi:10.1111/j.1365-3091.2005.00714.x.

Rouse, H. (1937), Modern conceptions of the mechanics of turbulence, *Trans. Am. Soc. Civ. Eng.*, *90*, 463–543.
Sklar, L. S., and W. E. Dietrich (2001), Sediment and rock strength controls on river incision into bedrock, *Geology*, *29*(12), 1087–1090, doi:10.1130/0091-7613(2001)029<1087:SARSCO>2.0.CO;2.
Sklar, L. S., and W. E. Dietrich (2004), A mechanistic model for river incision into bedrock by saltating bed load, *Water Resour. Res.*, *40*, W06301, doi:10.1029/2003WR002496.
Snyder, N. P., et al. (2003), Importance of a stochastic distribution of floods and erosion thresholds in the bedrock river incision problem, *J. Geophys. Res.*, *108*(B2), 2117, doi:10.1029/2001JB001655.
Stock, J. D., and D. R. Montgomery (1999), Geologic constraints on bedrock river incision using the stream power law, *J. Geophys. Res.*, *104*(B3), 4983–4993, doi:10.1029/98JB02139.
Tomkin, J. H., et al. (2003), Quantitative testing of bedrock incision models for the Clearwater River, NW Washington state, *J. Geophys. Res.*, *108*(B6), 2308, doi:10.1029/2001JB000862.
Tucker, G. E., and R. L. Bras (2000), A stochastic approach to modeling the role of rainfall variability in drainage basin evolution, *Water Resour. Res.*, *36*(7), 1953–1964, doi:10.1029/2000WR900065.
Turowski, J. M., et al. (2006), Experimental channel response to tectonic uplift, *J. Geophys. Res.*, *111*, F03008, doi:10.1029/2005JF000306.
Turowski, J. M., et al. (2007), Cover effect in bedrock abrasion: A new derivation and its implications for the modeling of bedrock channel morphology, *J. Geophys. Res.*, *112*, F04006, doi:10.1029/2006JF000697.
van der Beek, P., and P. Bishop (2003), Cenozoic river profile development in the Upper Lachlan catchment (SE Australia) as a test of quantitative fluvial incision models, *J. Geophys. Res.*, *108*(B6), 2309, doi:10.1029/2002JB002125.
Whipple, K. X. (2004), Bedrock rivers and the geomorphology of active orogens, *Annu. Rev. Earth Planet. Sci.*, *32*, 151–185, doi:10.1146/annurev.earth.32.101802.120356.
Whipple, K. X., and G. E. Tucker (1999), Dynamics of the stream-power river incision model: Implications for height limits of mountain ranges, landscape response timescales, and research needs, *J. Geophys. Res.*, *104*(B8), 17,661–17,674, doi:10.1029/1999JB900120.
Whipple, K. X., and G. E. Tucker (2002), Implications of sediment-flux-dependent river incision models for landscape evolution, *J. Geophys. Res.*, *107*(B2), 2039, doi:10.1029/2000JB000044.
Whipple, K. X., et al. (1999), Geomorphic limits to climate-induced increases in topographic relief, *Nature*, *401*(6748), 39–43, doi:10.1038/43375.
Wobus, C. W., et al. (2006), Self-formed bedrock channels, *Geophys. Res. Lett.*, *33*, L18408, doi:10.1029/2006GL027182.
Wong, M., G. Parker, P. DeVries, T. M. Brown, and S. J. Burges (2007), Experiments on dispersion of tracer stones under lower-regime plane-bed equilibrium bed load transport, *Water Resour. Res.*, *43*, W03440, doi:10.1029/2006WR005172.

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